Modelling of tree height growth

Comparison of different approaches

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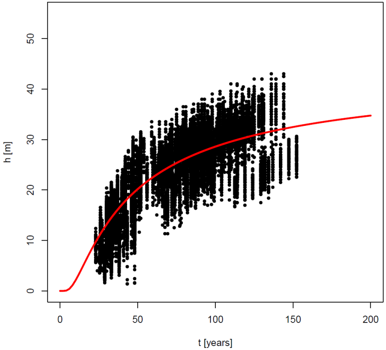
Model demands

$$h = f(t) + \varepsilon \quad (1)$$



- One inflection point
- Asymptote parallel to the abscissa
- Function through the origin (0, 0)
- Good numerical properties

Dataset

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- 23502 measurements
- 6339 trees
- 181 plots
- Norway spruce
- Lower saxony

Nonlinear models

Basic model (h = f(t), Sloboda 1971)

$$h_{ijk} = f(\psi_{ij}, t_{ijk}) + \epsilon_{ijk}$$

$$h_{ijk} = \psi_{ij}^{(1)} \left(\frac{\psi_{ij}^{(4)}}{\psi_{ij}^{(1)}} \right)^{exp\left[\frac{\psi_{ij}^{(2)}}{\left(\psi_{ij}^{(3)}-1\right) t_{ijk}^{\left(\psi_{ij}^{(3)}-1\right)}} - \frac{\psi_{ij}^{(2)}}{\left(\psi_{ij}^{(3)}-1\right) t_0^{\left(\psi_{ij}^{(3)}-1\right)}} \right]} + \epsilon_{ijk} \tag{2}$$

with

$$\psi_{ij} = (\psi_{ij}^{(1)}, ..., \psi_{ij}^{(4)})^T \tag{3}$$

(Nothdurft 2007, Schoneberg 2011)

- Plot i
- Tree j on plot i
- Measurement k on tree j on plot i

Modelling of the parameters

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$$\psi_{ij} = A \cdot \beta + B \cdot b_{ij} \tag{4}$$

$$\begin{aligned} \psi_{ij}^{(1)} &= \iota^{(1)} \\ \psi_{ij}^{(4)} &= \iota^{(4)} + r_i^{(4)} + r_{ij}^{(4)} \end{aligned} \tag{5}$$

- $\psi_{ij}^{(1)}$ free parameter
- $\psi_{ij}^{(4)}$ modelled using fixed and random effects

Introduction of covariates (fixed effects):

$$\begin{aligned} h_{ijk} &= f(\psi_{ij}, t_{ijk}, Temp_i) + \epsilon_{ijk} \\ \psi_{ij}^{(4)} &= \beta_{Temp} \cdot Temp_i + \iota^{(4)} + r_i^{(4)} + r_{ij}^{(4)} \end{aligned} \tag{6}$$

Linearisation approach

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Basic model (h = f(t), Korf function):

$$h = A \cdot exp(-Bt^{-C}) + \varepsilon \tag{7}$$

Linearised (if C fixed heuristically, Lappi 1997):

$$ln(h) = ln(A) - Bt^{-C} + \varepsilon \tag{8}$$

As mixed model:

$$ln(h_{ijk}) = ln(A_{ij}) - B_{ij} t_{ijk}^{-C} + \varepsilon_{ijk} \tag{9}$$

Modelling of covariates (not tested so far, D fixed heuristically):

$$ln(h_{ijk}) = ln(A_{ij}) - B_{ij} t_{ijk}^{-C} + \beta_{Temp} Temp_i^{-D} + \varepsilon_{ijk} \tag{10}$$

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Polynomial spline

$$h = Z\beta + \varepsilon \tag{11}$$

with

$$Z = \begin{bmatrix} 1 & t_1 & \dots & t_1^L & \begin{cases} (t_1 - k_1)^L, & k_1 \leq t_1 \\ 0, & else \end{cases} & \dots & \begin{cases} (t_m - k_1)^L, & k_m \leq t_1 \\ 0, & else \end{cases} \\ \vdots & & & & & & \vdots \\ 1 & t_n & \dots & t_n^L & \begin{cases} (t_n - k_1)^L, & k_1 \leq t_n \\ 0, & else \end{cases} & \dots & \begin{cases} (t_n - k_m)^L, & k_m \leq t_n \\ 0, & else \end{cases} \end{bmatrix} \tag{12}$$

Estimate β (least squares)

$$\hat{\beta} = (Z^T Z)^{-1} Z^T y \tag{13}$$

GAM – Model demands

- One inflection point (using less nodes)
- Asymptote parallel to abscissa
  (2 artificial heights at high ages based on Korf asymptote)
- Function through the origin (removing first column)

$$Z = \begin{bmatrix} t_1 & \dots & t_1^L & \begin{cases} (t_1 - k_1)^L, & k_1 \leq t_1 \\ 0, & else \end{cases} & \dots & \begin{cases} (t_m - k_1)^L, & k_m \leq t_1 \\ 0, & else \end{cases} \\ \vdots & & & & & \vdots \\ t_n & \dots & t_n^L & \begin{cases} (t_n - k_1)^L, & k_1 \leq t_n \\ 0, & else \end{cases} & \dots & \begin{cases} (t_n - k_m)^L, & k_m \leq t_n \\ 0, & else \end{cases} \end{bmatrix} \tag{14}$$

- Good numeric qualities (least squares)

All curves (nonlinear model)

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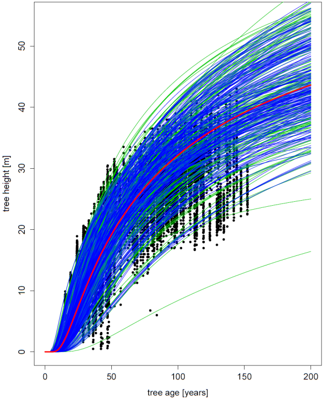

- Red: all random effects set to zero
- Blue: random effects on plot level
- Green: random effects on plot and tree level

Curves for one plot (nonlinear model)

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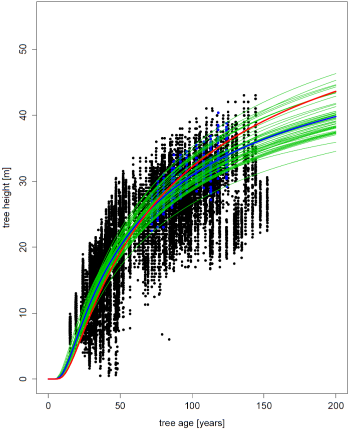

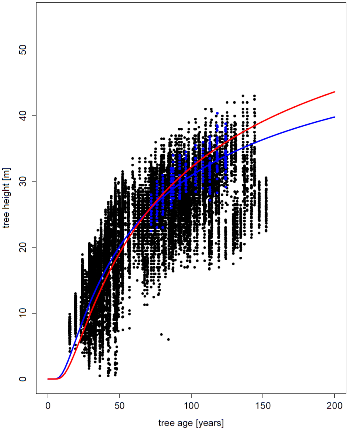

Curve for one tree (nonlinear model)

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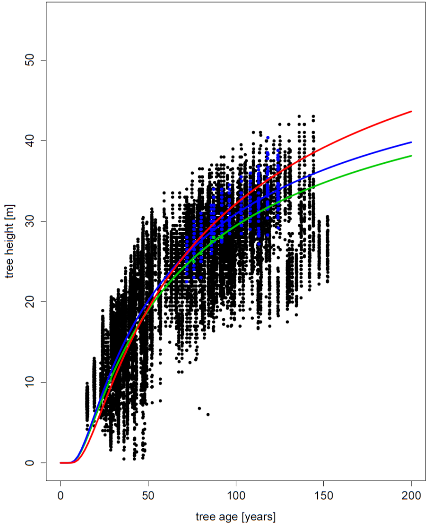

Residuals (nonlinear model)

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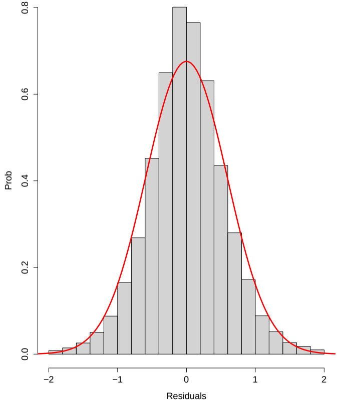

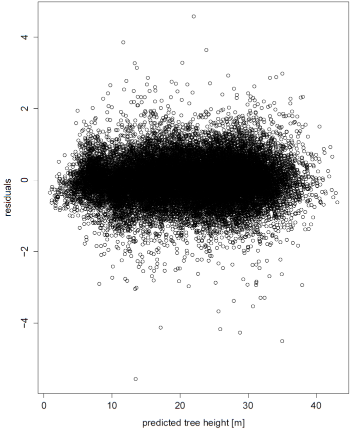

All curves (linearisation approach)

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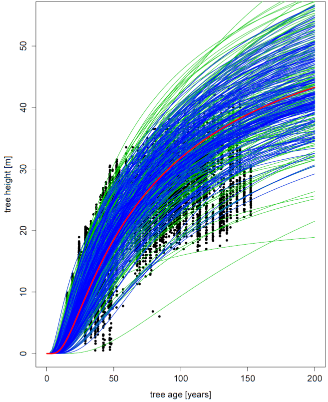

- $C = 0.757$ (heuristically)

Curves for one plot (linearisation approach)

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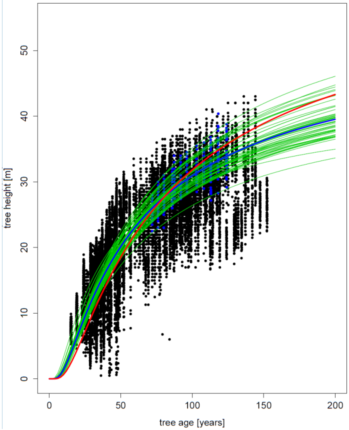
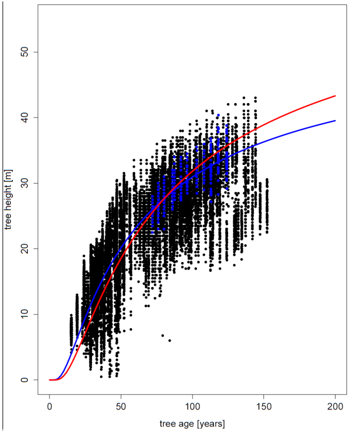
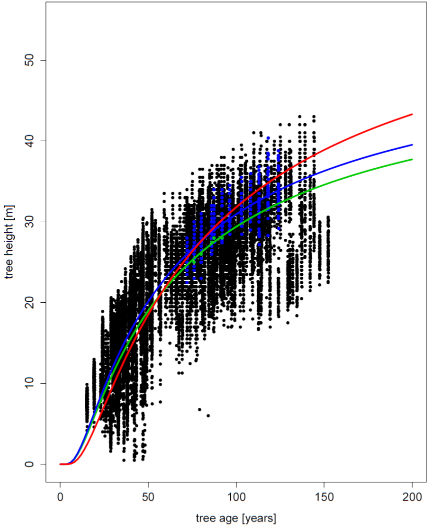
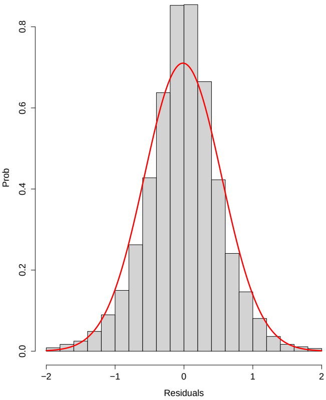


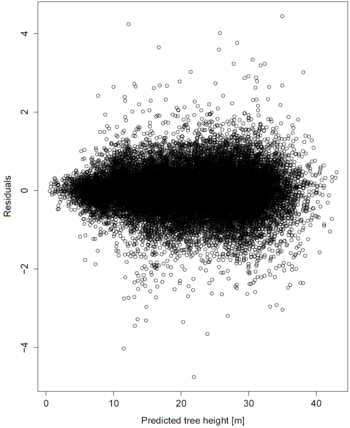


Curve for one tree (linearisation approach)



Residuals (linearisation approach, height scale)





Generalized additive model

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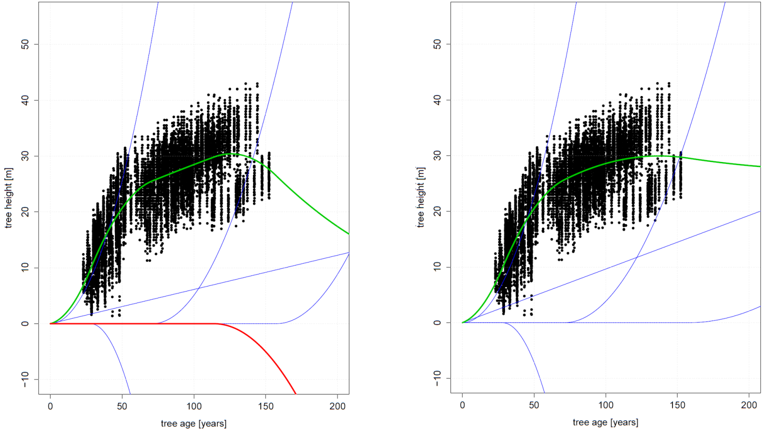

Thank you

for your attention!

Linearisation approach – Modelling of covariates

Temp influences *A*:

$$ln(h_{ijk}) = \iota + r_i + r_{ij} + \beta_{Temp} Temp_i^{-D} - B_{ij} t_{ijk}^{-C} + \varepsilon_{ijk} \quad (15)$$

Temp influences *B*:

$$ln(h_{ijk}) = ln(A_{ijk}) - (\iota + r_i + r_{ij} + \beta_{Temp} Temp_i) t_{ijk}^{-C} + \varepsilon_{ijk} \quad (16)$$

$$ln(h_{ijk}) = ln(A_{ijk}) - (\iota + r_i + r_{ij}) t_{ijk}^{-C} - \beta_{Temp} Temp_i t_{ijk}^{-C} + \varepsilon_{ijk} \quad (17)$$

References

Lappi, J. (1997). A longitudinal analysis of height/diameter curves. *Forest Science*, 43, 555–570. .II, 300.

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