Spatio-temporal prediction of site index based on forest inventories and climate change scenarios

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Motivation: Spatio-temporal prediction of site index

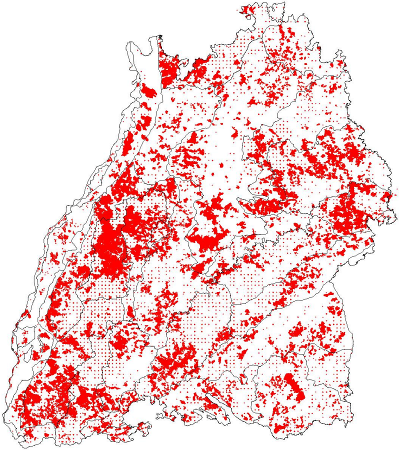
Definition (Site index)

A relative measure of forest site quality based on the height of the dominant trees at a reference age.

Goals:

- Reformulation of the existing site-productivity model of Schöpfer & Moosmayer (1972).
- Application of comprehensive data from regional and national forest inventories.
- To provide site-index predictions based on climate scenario data.

Dominant height measurements on forest inventory sample plots



Species	Sample plots
Spruce	91,642
Fir	33,987
Douglas-fir	9,356
Pine	31,559
Oak	28,230
Beech	81,241

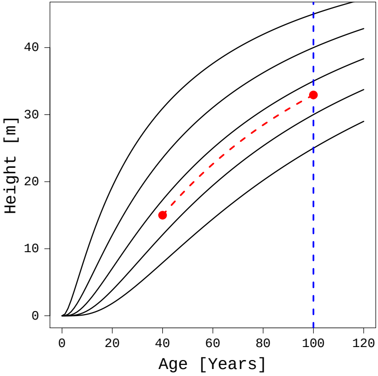
Climate data

We use

1. retrospective climate data for regression modeling,
2. climate scenario projections for spatio-temporal predictions.

Dominant height growth: Sloboda's (1971) differential equation

$$h\left(t_o|h(t),t\right) = \psi_1 \left(\frac{h(t)}{\psi_1}\right)^{\exp\left[\frac{\psi_2}{(\psi_3-1)t_o^{(\psi_3-1)}} - \frac{\psi_2}{(\psi_3-1)t^{(\psi_3-1)}}\right]}$$



The spatial process of site index

The spatial process of site index is defined for any arbitrary location s as

$$y(s) = x(s)'\beta + b(s) + \epsilon(s) \ ,$$

and is composed of

(i) a fixed linear trend dependent on site and climate variables in $x(s)$,

(ii) a spatially correlated error in terms of a Gaussian process $b(s)$, with 0 mean, variance θ^2, and spatial autocovariance function $c(h)$, h being the distance between two locations s_i and s_j,

(iii) and an unstructured error $\epsilon(s) \sim N\left(0, \sigma^2\right)$.

$\Rightarrow$ universal kriging (UK) model

Reformulation of the UK model as mixed model

The UK model can be reformulated as mixed model

$$y = X\beta + Ub + \epsilon$$

having covariance matrix

$$\mathbb{C}\mathrm{ov}(y) = \Sigma = \theta^2 URU' + \sigma^2 I_n$$

and correlation function

$$\mathbb{C}\mathrm{orr}\left[b(s_i), b(s_j)\right] = R = \rho(s_i - s_j) \ .$$

Spatial autocorrelation

The exponential correlation function

$$\rho\left(h\right) = \exp\left(-h/\alpha\right)$$

leads to the covariance function

$$c\left(h\right) = \begin{cases} \theta^2 \exp\left(-\frac{h}{\alpha}\right), & \text{for } h > 0 \\ \sigma^2 + \theta^2, & \text{for } h = 0 \end{cases}$$

and to the parametric semi-variogram model

$$\gamma\left(h\right) = \begin{cases} \sigma^2 + \theta^2 \left[1 - \exp\left(-\frac{h}{\alpha}\right)\right], & \text{for } h > 0 \\ 0, & \text{for } h = 0 \; . \end{cases}$$

Estimation: iteratively re-weighted generalized least squares (IRWGLS)

(I) Perform prior OLS $\hat{\beta}_{OLS} = (X'X)^{-1} X'y$ to estimate the mean model

(II) Iterate between (i) and (ii) until convergence is achieved

(i) Fit the parametric semi-variogram model to the empirical counterpart

$$\min \arg \sum_{u=1}^{K} \left\{2\gamma^{\sharp}(h(u)) - 2\gamma(h(u);\theta,\sigma,\alpha)\right\}^2$$

$$\hat{\gamma}^{\sharp}(h) = \frac{1}{2|N(h)|} \sum_{N(h)} [e(s_i) - e(s_j)]^2$$

$$N(h) \equiv \{(s_i;s_j) : s_i - s_j = h;\ i,j = 1,\ldots,n\}$$

$$\text{with } e = \begin{cases} y - X\hat{\beta}_{OLS}, & \text{in the first iteration} \\ y - X\hat{\beta}_{GLS}, & \text{otherwise} \end{cases}$$

(ii) Perform GLS $\hat{\beta}_{GLS} = \left(X'\hat{\Sigma}^{-1}X\right)^{-1} X'\hat{\Sigma}^{-1}y$

Estimation

Dilemma of large n

- Autocovariance matrix is high-dimensional and becomes intractable.
- Iteratively re-weighted generalized least squares (IRWGLS) is not feasible.

Hypotheses (given large data exists)

- Kriging of OLS residuals in moving 100 point neighborhoods leads to an unbiased spatial predictor.
- The error of the mean model can be neglected for constructions of prediction intervals.
- B-spline regression techniques provide robust estimates of biologically plausible cause-and-effect curves.

Prediction

Site-index predictions are spatially kriged at any location s_0 in moving 100 point neighborhoods by

$$\hat{y}_0 = x(s_0)'\hat{\beta}_{OLS} + \hat{c}_-'\hat{\Sigma}_-^{-1}\left(y_- - X_-\hat{\beta}_{OLS}\right) ,$$

"$-$" indicates the simplification of the universal kriging predictor in terms of a local predictor and $\hat{\Sigma}_-$ has dimension 100×100.
For interval prediction, the universal kriging variance

$$\begin{aligned}\mathbb{E}\left(y_0 - \hat{y}_0\right)^2 =& \sigma^2 + \theta^2 - c'\Sigma^{-1}c \\ &+ \left[x(s_0)' - c'\Sigma^{-1}X\right]\left[X'\Sigma^{-1}X\right]^{-1}\left[x(s_0)' - c'\Sigma^{-1}X\right]'\end{aligned}$$

is approximated by

$$\widehat{\mathbb{V}\mathrm{ar}}\left(\hat{y}_0\right) = \hat{\sigma}^2 + \hat{\theta}^2 - \hat{c}_-'\hat{\Sigma}_-^{-1} ,$$

in which the error of the estimator of β is neglected.
Constructions of 95%-intervals are obatained by

$$\hat{y}_0 \pm 1.96\sqrt{\widehat{\mathbb{V}\mathrm{ar}}\left(\hat{y}_0\right)} .$$

Mean model based on B-Spline regression techniques

A B-spline function is defined for the range of a vector of knots

$$\kappa = (\kappa_1, \ldots, \kappa_{l+1}, \ldots, \ldots, \kappa_{d-l}, \ldots, \kappa_d)^T \quad ,$$

and consists of d basis functions $\left(B_1^l(x), \ldots, B_d^l(x)\right)$ of degree l.
A single basis functions is defined for the range between $l+2$ consecutive knots and overlaps the $2l$ neighboring basis functions.
With given κ a basis function of degree $l = 0$ is defined by

$$B_v^0 = \mathbb{1}_{[\kappa_v, \kappa_{v+1})}(x) = \begin{cases} 1 & \text{for } \kappa_v \leq x < \kappa_{v+1} \\ 0 & \text{otherwise,} \end{cases} \qquad v = 1, \ldots, d-1 \ ,$$

and basis functions of higher order are recursively defined by

$$B_v^l(x) = \frac{x - \kappa_v}{\kappa_{v+l} - \kappa_v} B_v^{l-1}(x) + \frac{\kappa_{v+l+1} - x}{\kappa_{v+l+1} - \kappa_{v+1}} B_{v+1}^{l-1}(x) \ .$$

To guarantee that $\sum_{v=1}^{d} B_v^l(x) = 1$ holds for every $x \in [\kappa_1, \kappa_d]$, boundary knots are replicated l times, so that $\kappa_1 = \ldots = \kappa_{l+1}$ and $\kappa_{d-l} = \ldots = \kappa_d$.

Mean model based on B-Spline regression techniques

The entire regression matrix

$$X = (X_1, \cdots, X_p)$$

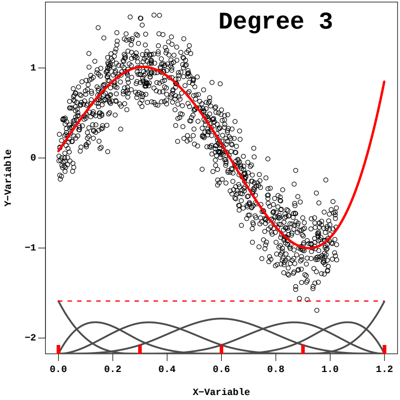
is composed of the p sub-matrices

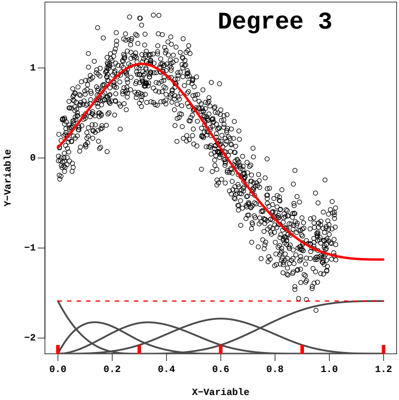
$$X_z = \begin{pmatrix} B_1^l(x_{z1}) & \dots & B_d^l(x_{z1}) \\ \vdots & & \vdots \\ B_1^l(x_{zn}) & \dots & B_d^l(x_{zn}) \end{pmatrix} .$$

Note: B-spline regression technique is nothing else than a multiple linear regression model!

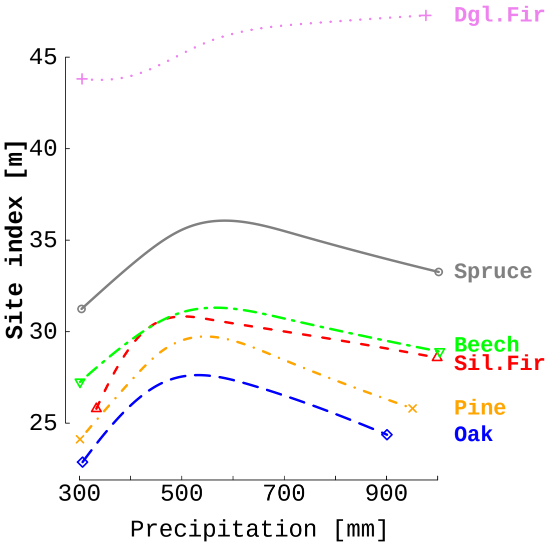
Mean model based on B-Spline regression techniques

Mean model based on B-Spline regression techniques and with natural boundary constraints.





Mean model based on B-Spline regression techniques



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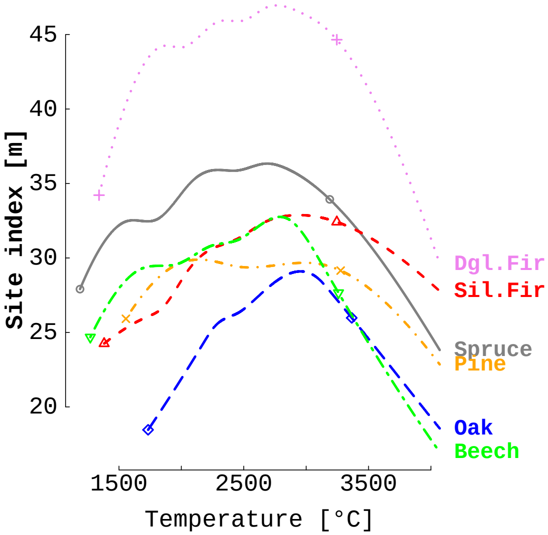
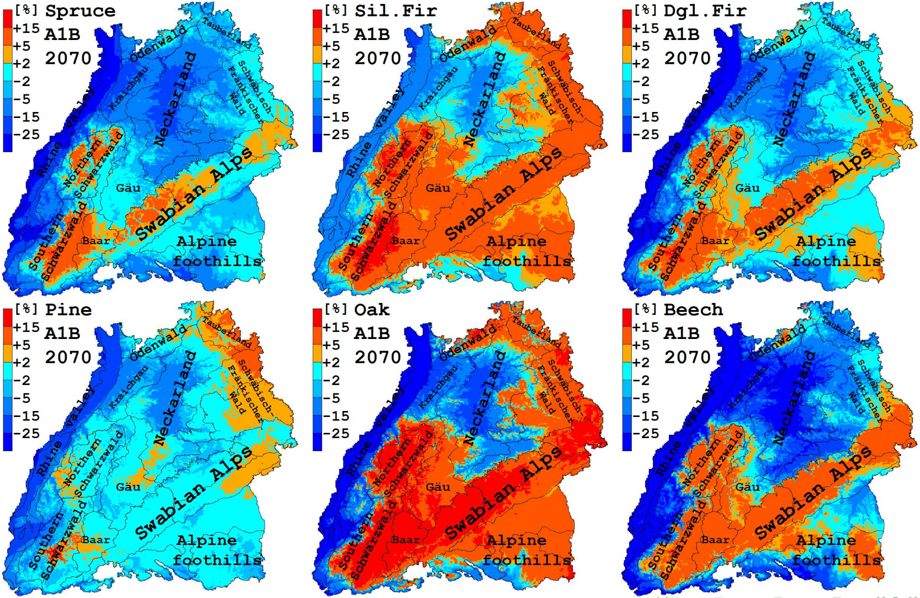
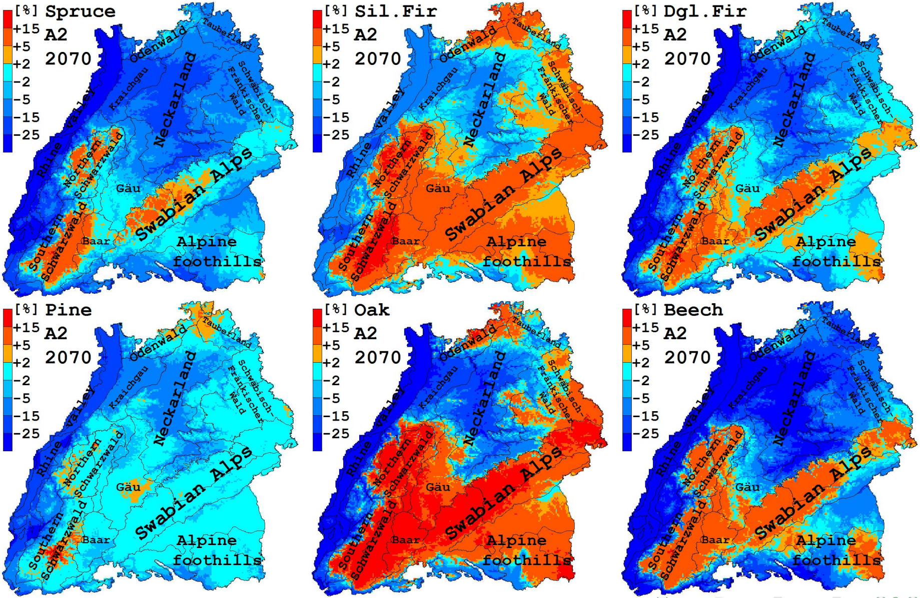


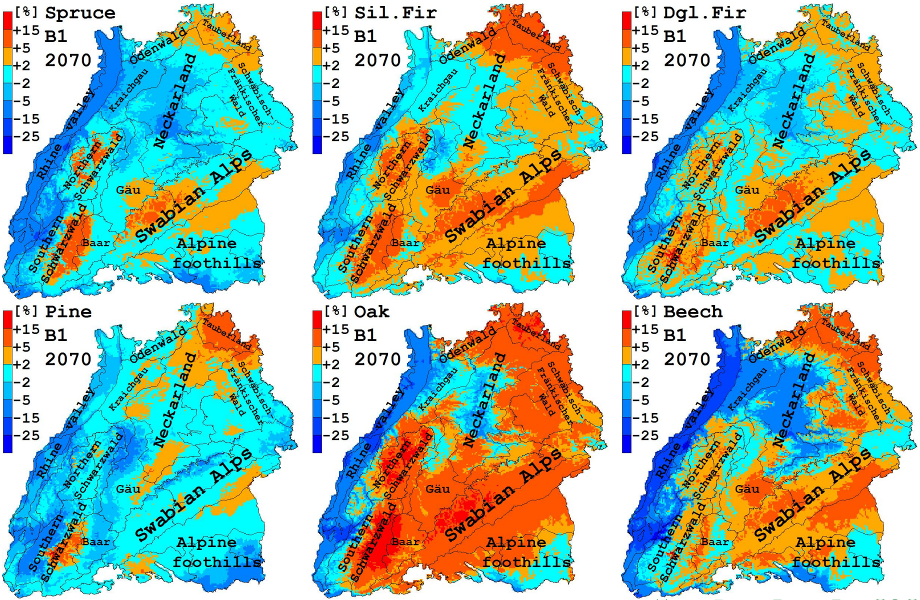
Predictions of relative changes of site index



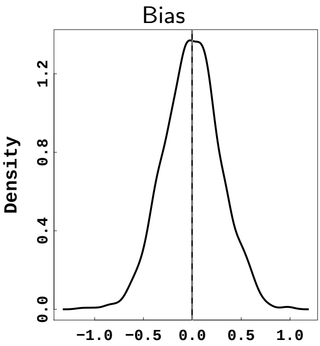
Predictions of relative changes of site index

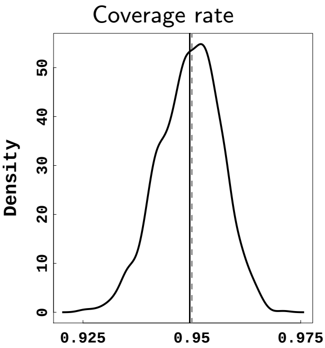


Predictions of relative changes of site index



Validation based on 1000 simulations of Gaussian random fields





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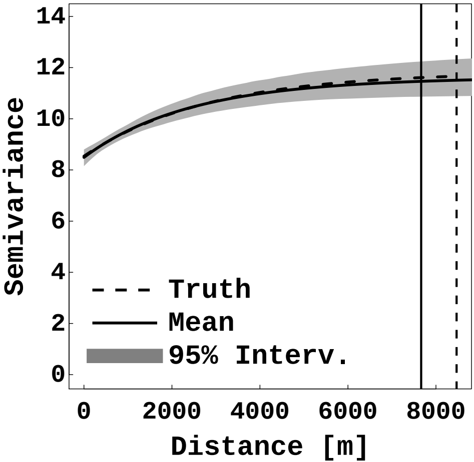

Résumé

- Site index decreases in low elevation areas and increases in mountainous regions.
- Silver fir and oak stands show increased site index also on lower elevation sites.
- Site index of Scots pine is less affected by a changing climate.
- Site conditions in the Alpine foothills region remain highly productive for growth of Norway spruce.

Résumé

If a universal kriging model is applied to a large forest inventory data set, then

- OLS is sufficient for the estimation of the mean and of the spatial covariance,
- spatial mean plus kriged OLS-residuals in moving 100 point neighborhoods is a practically unbiased predictor, and
- approximating the UK variance by neglecting the error of the mean yields a quasi-exact interval predictor.

Ende

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Thank you for your attentation !

Retrospective climate data

1. Lateral boundary conditions: NCAR/NCEP reanalysis series in resolution $2.5^\circ \times 2.5^\circ$ (lat./long.).
2. Dynamical downscaling in two steps: non-hydrostatic regional climate model (RCM) Weather Research and Forecasting (WRF) $\rightarrow$ 5 km $\times$ 5 km resolution.
3. Statistical downscaling: observations from the met-station network of the German National Meteorological Service $\rightarrow$ 50 m $\times$ 50 m resolution.
4. Bias correction and smoothing.

30-year long-term means (between 1978 and 2007) as regressor covariates:

- total precipitation during the growing season (PGS)
- total of average daily temperatures during the growing season (TGS)

Climate projection data

1. Existing runs of the hydrostatic RCM REMO on initiative of the German Federal Environment Agency
    - Lateral boundary conditions: general circulation model (GCM) ECHAM5-MPIOM.
    - $10\,\mathrm{km} \times 10\,\mathrm{km}$ horizontal grid resolution.
    - Scenarios A1B, A2 and B1 from Special Report on Emissions Scenarios (SRES).
2. Statistical downscaling $\rightarrow 1\,\mathrm{km} \times 1\,\mathrm{km}$: see retrospective climate data.
3. Spatio-temporal bias correction.